Analyzing irreversibility of heat transfer for a Non-Newtonian power-law flow with power-law temperature using differential transform method

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ABSTRACT

In the present paper, the differential transform method (DTM) with shooting method is applied to solve the non-Newtonian power-law flow problem. The governing partial differential equations are transformed to ordinary differential equations (ODE) with fractional order. Traditionally, the non-Newtonian power-law flow problem is numerically solved. However, numerical solution could not easily differentiate and integrate because the data is discrete. As such the DTM for the flow is presented. Using DTM, the analytic solution of the non-Newtonian power-law flow could be differentiable and integrated. The present method is more reliable and efficient in obtaining the numerical solutions that match well with those from the fourth-order Runge-Kutta method, which is considered close to the exact solution. Moreover, the effects of different parameters (suction number) on the irreversibility (such as $N_{sh}$, $N_{sf}$, $N_{ss}$) are analyzed and discussed.

Keywords: Power-law, entropy generation, irreversibility, differential transform, DTM.

INTRODUCTION

The non-Newtonian flow obviously was received by the scientist and engineer’s attention as a result of its importance in several applications such as mathematical and physical models. There were several research directions on these equations. Some studied the magnetohydrodynamic (MHD) viscous flows of non-Newtonian fluids (Makinde, 2015; Kavitha and Kishan, 2017; Makinde, 2015). For example, Makinde (2015) investigated the unsteady MHD Couette flow of an electrically conducting incompressible non-Newtonian third grade reactive fluid. The others studied the numerical solutions using meshfree method (Ahlikrona and Shcherbakov, 2017; Shamekhi and Sadeghy, 2009; Duan and Li, 2007). For example, Ahlikrona and Shcherbakov (2017) used meshfree to non-Newtonian free surface ice flow.

Recently, great attention had been paid to the analytic solutions (Liao, 2003; Barna et al., 2016; Ganji et al., 2011). For example, Liao (2003) used the analytic technique to give analytic solutions of magnetohydrodynamic viscous flows of non-Newtonian fluids over a stretching sheet. However, it can been seen that those numerical solutions do not have the characteristics of differentiation and integration because the numerical data is discrete. Hence, in order to resolve the issue, Puhov (1976) proposed the method of modified differential transform method (DTM). Since DTM is an extension of the Taylor series method, a numerical method can be used to obtain analytic solutions for problems of ordinary differential equations (ODEs) (Mirzaee, 2011; Chen and Ho, 1999) and partial differential equations (PDEs) problems (Jang et al., 2001; Etinkaya et al., 2011). Hence, in view of the aforementioned, a modified DTM will be used to obtain the numerical results for entropy generation in flow through a movable plate with variable temperature in the x-axis.

The important issue studied in the present article is
irreversibility analysis which is one of the energy conservation methods (Makinde and Anwar, 2010; Erbay et al., 2007; Makinde, 2010; Fang and Qun, 2011) for irreversibility that often arises in energy transfer processes (Aman and Nobari, 2011). Therefore, in order to study the irreversibility of a thermal system many scholars studied irreversibility due to heat convection. Bejan (1979, 1982) proposed the expression for the entropy generation rate, whose goal is to study entropy generation in a heat transfer process. Since then, many researchers studied the effect of various conditions on irreversibility of flow field.


Yazdi et al. (2014) proposed the second law analysis for Power-Law Fluid on Micro-patterned movable surface. For other interesting studies in solving heat transfer problem, Yang proposed integral transform so as to solve the heat transfer problems. The results indicated that the method is powerful for finding analytical solution for heat transfer equation.

PHYSICAL MODELING

Consider the same model as Zheng et al. (2012). Suppose the fluid is steady, incompressible, laminar and heat transfer over a flat surface, the present surface tension is power-law temperature gradients. The steady two-dimensional boundary layer equations of non-Newtonian fluid are:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\

t^0 + v \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y} \left( y \left( \frac{\partial}{\partial y} \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) \right) \\
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\partial}{\partial y} \left( \left( \frac{\partial}{\partial y} \left| \frac{\partial T}{\partial y} \right|^{n-1} \frac{\partial T}{\partial y} \right) \right)
\end{align*}
\]

(1)

(2)

(3)

With boundary conditions are:

\[ u(x, \infty) = v(x, 0) = 0 \]

\[ T(x, \infty) = T_n, T(x, 0) = T_\infty + Ax^{1/(2-n)} \text{ as } y \to \infty. \]

(4)

Where \( u, v \) are the velocity vectors in the \( x, y \) axis, respectively; \( T_n, T_\infty \) are constant temperature of the wall and ambient fluid.

Let:

\[ \eta = C_2 y, \; \psi = C_1 x^{2-n} f(\eta), \; t(x, y) = \frac{T - T_\infty}{T_n} \]

(5)

Then, Equations 1 to 3 will be transformed into:

\[ (f'' | f'' |^{|a-1} - f''^2 + f''') = 0 \]

\[ (| \theta |^{-n} \theta') + \frac{M (f \theta - f' \theta)}{2 - n} = 0 \]

(6)

The similarity boundary conditions are:

\[ f(0) = 0, f'(\infty) = 0, f'''(0) = -1, \theta(0) = 1, \theta(\infty) = 0. \]

(7)

Let \( u = f, v = f', w = f'' , y = \theta, z = \theta' \) then Equations 6 to 7 are solved to be an initial value problems of a system:

\[ u' = v, u(0) = 0 \]

\[ v' = w, v(0) = p \]

\[ w' = \frac{n}{n |w|^{1-n} v^2} - \frac{|w|^{1-n} w u}{n}, w(0) = -1 \]

\[ y' = z, y(0) = 1 \]

\[ z' = -\frac{|z|^{n-1} M}{n(2 - n)} (uz - vy), z(0) = q \]

(8)

Equations 6 to 7 are not a standard initial value ODE; it is difficult to get the numerical solution. We delete the boundary values \( v(\infty), y(\infty) \) and add the initial value \( v(0) = p, y(0) = q \). Then, we can find the values \( p, q \) that the solution of the system (8) go through \( v(0) = p, y(0) = q \) and satisfy the boundary conditions \( v(\infty) = y(\infty) = 0 \) by shooting method.
ANALYSIS FOR ENTROPY GENERATION

For a flow through a moving plate with variable temperature, the local entropy generation rate of the irreversibility of a thermal system is defined as:

\[
S'''' = \frac{k}{T_x^2} \left( \frac{\partial T}{\partial y^2} \right) + \frac{\mu}{T_x} \left( \frac{\partial u}{\partial y} \right)^2
\]

(9)

Bejan (1979, 1982) presented that the ratio of the local entropy generation rate to the characteristic entropy generation rate is the same for the entropy generation number. As such the entropy generation number form (6) can be:

\[
NsRe_x = \left( \frac{d\theta}{d\eta} \right)^2 + \frac{Pr Ec}{\Omega} \left( \frac{d^2 f}{d\eta^2} \right)^2 = N_{sh} + N_{sf}
\]

(10)

Where the Eckert number \( Ec = U_\infty / (T_w - T_\infty) \) and dimensionless temperature difference \( \Omega = (T_w - T_\infty) / T_\infty \). The characteristic entropy generation is \( S_0'' = k(T_w - T_\infty)U_\infty^2 / (T_\infty^2 \nu^2) \), the heat transfer irreversibility is \( N_{sh} \) and the fluid friction irreversibility is \( N_{sf} \). As such Bejan number (\( Be \)) (Bejan, 1979, 1982) is the same as:

\[
Be = \frac{1}{1 + \phi}
\]

(11)

Where \( \phi = \frac{N_{sf}}{N_{sh}} \)

MODIFIED TECHNIQUE DESCRIPTION

Now, consider a brief description of standard DTM. Let \( v(t) \) be an analytic function in a domain \( D \). Here, \( t = a \) represent any point in \( D \). Then, Taylor series expansion for the function of \( v(t) \) is defined as:

\[
v(t) = \sum_{k=0}^{\infty} \frac{(t-a)^k}{k!} \left[ \frac{d^k v(t)}{dt^k} \right]_{t=a}, t \in D.
\]

It is worth mentioning that the function of \( v(t) \) can be a Maclaurin series when \( a = 0 \):

\[
v(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[ \frac{d^k v(t)}{dt^k} \right]_{t=0}, t \in D.
\]

The differential transformed function \( v(t) \) is expressed as:

\[
V(k) = \frac{H^k}{k!} \left[ \frac{d^k v(t)}{dt^k} \right]_{t=0}, k = 0, 1, 2, \ldots
\]

Where \( V(k) \) can be the transformed function and \( v(t) \) is the original function. The differential spectrum of \( V(k) \) is confined in the interval \( t \in [0, H] \); \( H \) is the given constant number. The differential inverse transform of \( V(k) \) is defined as:

\[
v(t) = \sum_{k=0}^{\infty} \left( \frac{I}{H} \right)^k V(k), t \in D.
\]

In practical applications, because the differential transformation method is derived from the Taylor series expansion, it is found that the number of arguments required to restore the unknown function precisely can be reduced by specifying an appropriate value of the constant \( H \). In other words, the function \( v(t) \) can be obtained in terms of a finite series as:

\[
v(t) = \sum_{k=0}^{n} \left( \frac{I}{H} \right)^k V(k), t \in D.
\]

Next, we state some important properties of the Taylor differential transformation derived using the aforementioned expressions which are needed in the sequel.

The operation properties of differential transformation

If the transformed functions for both \( U(k) \) and \( V(k) \) are derived from \( u(x) \) and \( v(x) \), respectively, then, the fundamental mathematical operations of differential transformation are listed as:

1. If \( z(t) = u(t) \pm v(t) \) then \( Z(k) = U(k) \pm V(k) \).
2. If \( z(t) = au(t) \) then \( Z(k) = aU(k) \).
(3) If \( z(t) = \frac{d^n u}{dt^n} \), then
\[
Z(k) = (k + 1)(k + 2) \cdots (k + m)U(k + m).
\]
(4) If \( z(t) = u(t)v(t) \), then
\[
Z(k) = U(k) \otimes V(k) = \sum_{i=0}^{k} U(i)V(k - i).
\]

Let \( w(t) = W(0) + \sum_{k=1}^{n} t^k W(k) \). By binomial theorem:
\[
|w|^{1-n} = |W(0)|^{1-n} \left[ |W(0)|^{1-n} + (1-n) \right]^{\frac{1}{n}} \sum_{k=1}^{n} t^k W(k) + (1-n)(1-n-1)/2 \left| \sum_{k=1}^{n} t^k W(k) \right|^2 + \]
\[
= Q(0) + \sum_{k=1}^{n} t^k Q(k)
\]
(12)

Putting (3) and (12 to 13) in (8), the transform of (8) becomes:
\[
(k + 1)U(k + 1) = V(k), U(0) = 0
\]
\[
(k + 1)V(k + 1) = W(k), V(0) = p
\]
\[
(k + 1)W(k + 1) = \frac{Q(k)}{n} \otimes [V(k) \otimes V(k) - U(k) \otimes W(k)], W(0) = -1
\]
\[
(k + 1)Y(k + 1) = Z(k), Y(0) = 1
\]
\[
(k + 1)Z(k + 1) = -\frac{MQQ(k)}{n(2-n)} \otimes [U(k) \otimes Z(k) - V(k) \otimes Y(k)], Z(0) = \eta
\]
(14)

Actually, DTM could not be used for large interval or global solutions. DTM only can get the solution for small interval. For large interval, the accuracy of numerical solution is not perfect. So we make many solutions of the system (14) for small intervals. Furthermore, we combine these small intervals. For initial value point 1, we have the solution (1A), \( 0 \leq t \leq h \). Using point A as an initial value, then we have the solution (AB). Continuing this process, then, we have the global solutions. In the present article, let \( h = 2 \).

RESULTS AND DISCUSSION
The algorithms are coded in the computer algebra package-Mathematica. The environment variable digit controlling the number of significant digits is set to 32 in all the calculations done in this paper. The fourth-order Runge-Kutta method which is considered the “exact” solution is compared to the present method (Tables 1 and 2) in case 1 given as:

**Case 1:** Consider the case, \( n = 0.8, M = 3, Pr = 0.7 \).

The irreversibility analysis is presented for variable temperature for different Power-law number. Similarity solutions, such as stream function, velocity-gradient, shear stress, temperature-gradient, heat transfer irreversibility \( \eta \), fluid friction irreversibility \( N_0 \), entropy generation number \( N_{ae} \) and Bejan number \( Be \), can be obtained by DTM. For Figure 1, stream function becomes higher as Power-law number decreases. Figure 2 exhibits the effect of the Power-law number. Velocity-gradient decreases quickly as the Power-law number increases. The maximum value is on the end point 0, so it is the interface velocity increases as the velocity boundary layer decreases. Figure 3 shows that the minimum value is on the edge point 0. The shear stress is non-positive and convergent to 0 more quickly as the Power-law number increases. As such, we have the same conclusion (Zheng et al., 2012) that the thickness of velocity boundary layer is thinner for the Power-law number increase. Figure 4 exhibit that the temperature decreases more quickly as the Power-law number increases. Figures 5 and 6 show that the effect of \( N_{sh} \) is greater than the \( N_{sf} \) in the development of irreversibility. Also, \( N_{sh} \) and \( N_{sf} \) curves drop quickly near \( \eta = 0 \), after which the \( N_{sh} \) and \( N_{sf} \) curves develop in the form of horizontal asymptote as \( \eta \) increases, indicating that increasing the value of \( \eta \) leads to decreasing the effects of irreversibility in the present model. Figure 7 shows the maximum value of \( N_{ss} \) appears near \( \eta = 0 \), and developed in the form of horizontal asymptote with increasing \( \eta \). For Bejan number \( Be \), as be seen in
Table 1: The present method $f, f', f''$ iterations $m = 10$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\eta$</th>
<th>Rk4</th>
<th>Present method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
<td>0.619</td>
<td>0.623</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.871</td>
<td>0.888</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.988</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.080</td>
<td>1.102</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>1.116</td>
<td>1.121</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.116</td>
<td>1.121</td>
</tr>
</tbody>
</table>

|          | 8      | 0.003 | 0.004         |
| $f''$    | 9      | 0.001 | 0.002         |
|          | 10     | 0.000 | 0.000         |

|          | 8      | 0.002 | 0.003         |
| $f'''$   | 9      | 0.001 | 0.001         |
|          | 10     | 0.000 | 0.000         |

Table 2: The present method $\theta, \theta'$ iterations $m = 10$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\eta$</th>
<th>Rk4</th>
<th>Present method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>0.148</td>
<td>0.151</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0301</td>
<td>0.0310</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.00883</td>
<td>0.00895</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.00336</td>
<td>0.00344</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.00151</td>
<td>0.00159</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.00029</td>
<td>0.00031</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.00020</td>
<td>0.00022</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

| $\theta'$ | 7      | -0.00023 | -0.00024 |
|           | 8      | -0.00012 | -0.00013 |
|           | 9      | 0.0000  | -0.00001 |
|           | 10     | 0.0000  | 0.0000   |

Figure 8, which takes into account whether the fluid friction irreversibility dominates or the heat transfer for the present model. This result indicates the maximum value of $\text{Be}$ appears near $\eta = 0$ and thereafter develop in the form of horizontal asymptote with increasing $\eta$. This fact shows that entropy generation near the leading edge is dominated by $N_{sh}$, however, entropy generation is dominated by $N_{sf}$ when developing in the form of horizontal asymptote with increasing $\eta$.

CONCLUSION

The study of entropy generation is important because
energy conservation cannot be ignored in many practical applications. In order to study irreversibility of a moving plate with various temperatures, suitable similarity variables are used to transform the local entropy...
**Figure 3:** Shear stress distribution for different Power-law number, normal line \( n = 0.6 \); bold line \( n = 0.8 \); points are \( n = 1 \).

**Figure 4:** Temperature distribution for Power-law number, normal line \( n = 0.6 \); bold line \( n = 0.8 \); points are \( n = 1 \).
Figure 5: Nsh profiles for different Power-law number, normal line n = 0.6; bold line n = 0.8; points are n = 1.

Figure 6: Nsf profiles for different Power-law number, normal line n = 0.6; bold line n = 0.8; points are n = 1.
Figure 7: Nsx profiles for different Power-law number, normal line $n = 0.6$; bold line $n = 0.8$; points are $n = 1$.

Figure 8: Bejan number for different Power-law number, normal line $n = 0.6$; bold line $n = 0.8$; points are $n = 1$. 
generation rate to entropy generation number. A modified differential transform method (DTM) with shooting method is used to obtain the similarity solution of the entropy generation, the proposed numerical method; their accuracy of analytic solution is very high. The similarity solution profiles $N_{slp}$, $N_{sf}$ and $N_{sx}$ have a maximum value near a point $n=0$. The slope of similarity solution profiles will drop down at the edge of the boundary layer.

REFERENCES


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